

The Influence of Hull Deflection on Double Bottom Tank Measurement

Introduction

Deflection of the hull, i.e. deformation of the bottom of the ship from a straight line to a curved line either concave (sag +) or convex (hog -) is determined from the following formula:

$$d = T_K - \frac{T_{KF} + T_{KA}}{2} \quad (1)$$

where: T_K – keel draught at midship*;

T_{KF} – keel draught at fore perpendicular (FP);

T_{KA} – keel draught at aft perpendicular (AP).

Ship's hull deflection is well known and taken into account during draught survey.

There are three reasons for ship's hull deformation namely:

- a constant deflection caused during hull building or improper exploitation ;
- improper weight distribution alongside ship;
- thermal (solar) deflection caused by different temperatures of sea water and air (specially during sunny days when the deck temperature increased considerably).

The solar deflection may be significant. In [1] a case was described when a 70 000 DWT vessel laying at anchor alters her deflection from sag 5 cm (at 0700) to hog 27,5 cm (at 1700). The ship's hull deflection is taken under consideration during displacement calculation while draft survey is performed. There arise a question if considerable hull deflection influences the correct measurement of double bottom tanks.

Hull deflection equation

The exact curve of hull deflection is unknown, but for the practical purposes it is assumed that it takes a parabolic form [1,2]:

$$y = a \cdot x^2 + b \cdot x + c \quad (1)$$

where: y – ordinate (draught);

x – abscissa (longitudinal coordinate of draught marks).

To determine the parabola coefficients a , b and c three equations are needed:

$$\begin{aligned} T_{KAR} &= a \cdot x_A^2 + b \cdot x_A + c \\ T_{KR} &= a \cdot x_M^2 + b \cdot x_M + c \\ T_{KRF} &= a \cdot x_F^2 + b \cdot x_F + c \end{aligned} \quad (2)$$

Where: T_{KAR} – keel draught reading at aft draught mark;

* All symbols and sign convention used in this paper are according to the proposals stated in the primary drafting of the MCS Cir. 920. In the later formulation of this circular some suggestions relating to sign convention are omitted.

T_{KR} – keel draught reading at mid draught mark;
 T_{KFR} – keel draught reading at fore draught mark;
 x_A – distance of aft mark from aft perpendicular (AP);
 x_M – distance of mid mark from AP;
 x_F – distance of fore mark from AP.

To solve this system of equations matrix calculation was used and four determinant were defined as follows:

$$Det|W| = \begin{vmatrix} x_A^2 & x_A & 1 \\ x_M^2 & x_M & 1 \\ x_F^2 & x_F & 1 \end{vmatrix} \quad Det|a| = \begin{vmatrix} T_{KAR} & x_A & 1 \\ T_{KR} & x_M & 1 \\ T_{KFR} & x_F & 1 \end{vmatrix}$$

$$Det|b| = \begin{vmatrix} x_A^2 & T_{KAR} & 1 \\ x_M^2 & T_{KR} & 1 \\ x_F^2 & T_{KFR} & 1 \end{vmatrix} \quad Det|c| = \begin{vmatrix} x_A^2 & x_A & T_{KAR} \\ x_M^2 & x_M & T_{KR} \\ x_F^2 & x_F & T_{KFR} \end{vmatrix}$$

Using the determinant all the coefficient may be determined:

$$a = \frac{Det|a|}{Det|W|} \quad b = \frac{Det|b|}{Det|W|} \quad c = \frac{Det|c|}{Det|W|}$$

Now an equation may be written to determine draught for any point at abscissa x_p :

$$T_p = a \cdot x_p^2 + b \cdot x_p + c \quad (3)$$

Using this equation once for aft perpendicular (AP) $x_A=0$ and again for fore perpendicular (FP) $x_F=LBP$ (length between perpendiculars) trim of the vessel between perpendiculars may be determined (negative sign is attributed to trim by stern [4]):

$$t = T_{KF} - T_{KA}$$

and mean draught:

$$T_{KM} = \frac{T_{KF} + T_{KA}}{2}$$

If the draught at mid (T_K) is different from T_{KM} then it means that a hull deflection exists. Ship's deflection (sag or hog) is usually calculated using following formula:

$$d = T_K - T_{KM}$$

Sometime the deflection is considerably and cause that the calculated trim between perpendiculars is not constant along the ship's hull. To prove it let us solve an example.

Example: A vessel: $LBP = 110,0$ m; $T_{KAR} = 3,50$ m; $x_A = 3,5$ m; $T_{KR} = 3,18$ m; $x_M = 55$ m; $T_{KFR} = 3,10$ m; $x_F = 107$ m

Tank location: from $x_a = 74$ m from AP to $x_f = 95$ m.

Calculate: trim between perpendiculars, hull deflection, draught on coordinates x_a and x_f , local trim in the tank range.

Salvation: using the described matrix calculations the following results were received:
Trim between perpendiculars $t = -0,428$ (by stern); deflection = $-0,137$ m (hogging); draught on coordinates $x_a - T_a = 3,12$ m; $x_f - T_f = 3,097$ m. Local trim for tank range:

$$t_l = \frac{T_f - T_a}{x_f - x_a} \cdot LBP = \frac{3,097 - 3,122}{95 - 74} \cdot 110 = -0,131 \text{ m}$$

As from this example may be seen, hull deformation cause alteration of trim which should be applied during reading the liquid volume from sounding table from $-0,43$ to $-0,13$ m.

Local trim for tank range

Knowing the coefficient of the parabola equation it is possible to determine the local trim for any point along the hull as the derivative of this equation is equal to the tangent of trim angle:

$$\tan \psi = (a \cdot x_p^2 + b \cdot x_p + c)' \quad (4)$$

If the coordinate x_p will present the centre of a tank then the local trim (t_l) will be:

$$t_l = \tan \psi \cdot LBP \quad (5)$$

Assuming that the deflection curve is a parabola the above shown calculation are mathematically correct but a bit complicated and time consuming.

If we assume that the fore, mid and aft draughts refer to perpendiculars and additionally that $LBP = 1$; $x'_F = 1$; $x'_M = 0,5$ and $x'_{AP} = 0$ (x' -relative coordinate)[5] then:

$$\begin{aligned} T_{KA} &= a \cdot 0^2 + b \cdot 0 + c \\ T_{KR} &= a \cdot 0,5^2 + b \cdot 0,5 + c \\ T_{KF} &= a \cdot 1^2 + b \cdot 1 + c \end{aligned} \quad (6)$$

Solving this system of equations we get:

$$\begin{aligned} a &= 2 \cdot T_{KA} - 4 \cdot T_K + 2 \cdot T_{KF} \\ b &= 4 \cdot T_K - 3 \cdot T_{KA} - T_{KF} \\ c &= T_{KA} \end{aligned} \quad (7)$$

Using these coefficients, draught for any point along the hull may be determined but the abscissa must be presented as relative length $x' = x/LBP$. This way though shorter is still time consuming. The shortest way to calculate the local trim for a given tank is to make use of the trim between perpendiculars and determine only the alteration of trim caused by hull deflection. Assuming that a vessel is on even keel and additionally assuming that the fore and aft draughts are equal zero and at midship the draught is d (deflection) we may determine the a , b and c coefficients using (7):

$$\begin{aligned} a &= 2 \cdot 0 - 4 \cdot d + 2 \cdot 0 = -4 \cdot d \\ b &= 4 \cdot d - 3 \cdot 0 - 0 = 4 \cdot d \\ c &= 0 \end{aligned}$$

Now the parabola equation will be;

$$y = -4 \cdot d \cdot x'^2 + 4 \cdot d \cdot x'$$

The derivative of this equation is equal the tangential at point x' :

$$\tan \psi = y' = (-4 \cdot d \cdot x'^2 + 4 \cdot d \cdot x')' = -8 \cdot d \cdot x' + 4 \cdot d \quad (8)$$

and as relative length of the vessel is $LBP=1$ the correction for local trim is:

$$\Delta t_l = \tan \psi$$

Transforming equation (8) we get:

$$\Delta t_l = -8 \cdot d \cdot \left(x' - \frac{1}{2} \right) \cdot \frac{LBP}{LBP} = -8 \cdot d \cdot \frac{x' \cdot LBP - \frac{LBP}{2}}{LBP} = -8 \cdot d \cdot \frac{x - \frac{LBP}{2}}{LBP}$$

$$\Delta t_l = -8 \cdot d \cdot \frac{x_{\square}}{LBP} \quad (9)$$

Where: x' – relative longitudinal distance of tank centre from AP;
 x_{\square} – real longitudinal distance of tank centre from amidships.

Using equation (8) a diagram was performed for the x' in range from 0 to 1.

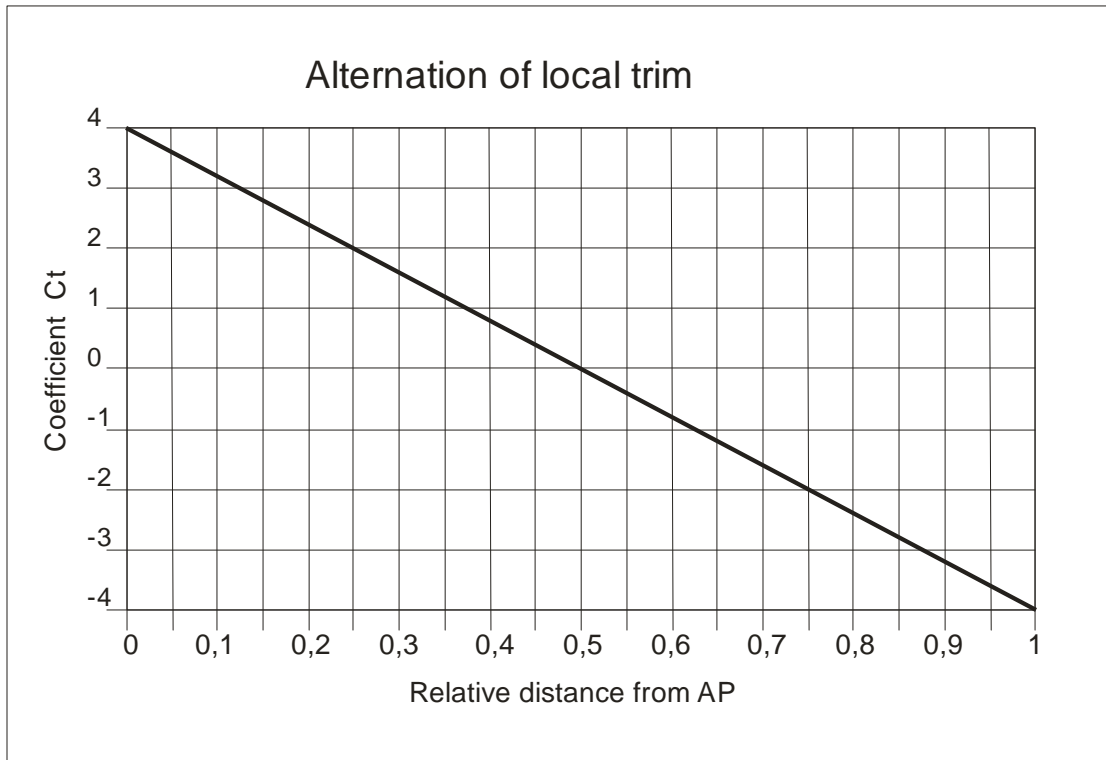


Fig. 1. Diagram of the trim correcting factor C_t

Using the trim correcting factor C_t the value of trim alternation Δt_l may be calculated as follows:

$$\Delta t_l = d \cdot C_t \quad (10)$$

And finally the local trim for the tank:

$$t_l = t + \Delta t_l \quad (11)$$

Example: For the same data as in the pervious example calculate the local trim for the tank using the trim correcting coefficient C_t . All calculations should follow algebraic rules regarding signs.

Solution. Centre of the tank:

$$C_G = \frac{74 + 95}{2} = 84,5 \text{ m}$$

Relative distance of the centre of tank:

$$x' = \frac{C_G}{LBP} = \frac{84,5}{110} = 0,77$$

Trim correcting coefficient from diagram = -2,15

$$\Delta t_l = d \cdot C_t = (-0,14) \cdot (-2,15) = 0,30$$

Local trim for the tank:

$$t_l = t + \Delta t_l = -0,43 + 0,30 = -0,13 \text{ m}$$

All calculation should follow algebraic rules regarding signs. Negative sign is attributed to trim by stern.

Additionally tanks situated direct on bottom alter their shape and cause inaccuracy in the measurement of the liquid volume (fig. 2).

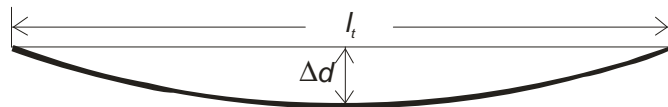


Fig. 2. Bottom deflection Δd in range of one tank.

Along the tank length exist deflection which is a small part of the hull deflection and it is determinate by:

$$\Delta d = d \cdot \left(\frac{l_t}{LBP} \right)^2 \quad (12)$$

To prove this formula we may use the relative parabola equation.

$$d = T_K - \frac{T_{KF} + T_{KA}}{2}$$

Substituting each value on the right side of this equation by it's parabola equation we get:

$$\begin{aligned} d &= a \cdot \left(\frac{x'}{2} \right)^2 + b \cdot \frac{x'}{2} + T_{KA} - \left(\frac{a \cdot x'^2 + b \cdot x' + T_{KA} + a \cdot 0 + b \cdot 0 + T_{KA}}{2} \right) = \\ &= \frac{2 \cdot a \cdot \left(\frac{x'}{2} \right)^2 + 2 \cdot b \cdot \frac{x'}{2} + T_{KA} - a \cdot x'^2 + b \cdot x' + 2 \cdot T_{KA}}{2} = a \cdot \frac{x'^2}{4} - a \cdot \frac{x'^2}{2} = -\frac{a}{4} \cdot x'^2 \end{aligned}$$

As it looks from the final result the total hull deflection is equal one quarter of the constant a with opposite sign multiplied by square of the relative length if ($x'^2=1$):

$$d = -\frac{a}{4} \quad (13)$$

Similarly for practical purposes:

$$\Delta d = d \cdot \left(\frac{l_t}{LBP} \right)^2 \quad (14)$$

The additional solid cause by the deflection in the tank range may be calculated by multiplying the tank length (l_t) by the side plane formed by the parabola and a straight line (fig. 2). Using Simpson formula we get the side plane:

$$F = \frac{l_t}{2 \cdot 3} \cdot (0 + 4 \cdot \Delta d + 0) = \frac{2 \cdot \Delta d \cdot l_t}{3} \quad (15)$$

And finally the addition volume:

$$\Delta V_d = F \cdot b_t = d \cdot \left(\frac{2 \cdot l_z^3 \cdot b_t}{3 \cdot LBP^2} \right) = d \cdot C_d \quad (16)$$

The easiest way to calculate this correction is to multiply the hull deflection with appropriate sign (sag +, hog -) by coefficient C_d once calculated for a given tank.

Formula (16) is true only for tanks placed in parallel middle body of the vessel.

Example

Ships data: $LBP = 185$ m; draft even keel ; hull deflection $d = 0,15$ m (sag)

Two bottom tanks containing fuel oil: length = 17,1 m; breadth = 6,5 m; high = 1,68 m;

$LCG = 41,4$ m; sounding $PS = 0,98$ m; $SB = 0,92$ m.

Sounding pipe are situated at the aft bulkhead. Calculate the volume of fuel of non deflected and deflected vessel.

Solution.

Non deflected vessel

$$v_{PS} = l_t \cdot b_t \cdot sounding = 17,1 \cdot 6,5 \cdot 0,98 = 108,927 m^3$$

$$v_{SB} = l_t \cdot b_t \cdot sounding = 17,1 \cdot 6,5 \cdot 0,92 = 102,258 m^3$$

Deflected vessel

Local trim coefficient:

$$C_t = -8 \cdot \left(\frac{LCG - \frac{LBP}{2}}{LBP} \right) = -8 \cdot \left(\frac{41,4 - 92,5}{185} \right) = 2,21$$

$$\Delta t_l = d \cdot C_t = 0,15 \cdot 2,21 = 0,33 m \text{ (by head)}$$

Sounding correction (wage formula):

$$\Delta s = \frac{l_t \cdot \Delta t_l}{2 \cdot LBP} = \frac{17,1 \cdot 0,33}{2 \cdot 185} = 0,015 m$$

$$v_{PS_c} = l_t \cdot b_t \cdot sounding \text{ corrected} = 17,1 \cdot 6,5 \cdot 0,995 = 110,594 m^3$$

$$v_{SB_c} = l_t \cdot b_t \cdot sounding \text{ corrected} = 17,1 \cdot 6,5 \cdot 0,935 = 103,925 m^3$$

$$\Delta V_d = d \cdot \left(\frac{2 \cdot l_z^3 \cdot b_t}{3 \cdot LBP^2} \right) = 0,15 \cdot \left(\frac{2 \cdot 17,1^3 \cdot 6,5}{3 \cdot 185^2} \right) = 0,095 m^3$$

True volume:

$$\text{Port tank} = 110,594 + 0,095 = 110,689 m^3$$

$$\text{Starboard tank} = 103,925 + 0,095 = 104,020 m^3$$

(Difference $3,524 m^3$)

Conclusions

1. Considerable ship's hull deflection cause inaccuracy in determining the volume of liquids in bottom tanks.
2. There are two sources of this inaccuracy: first is the alteration of trim in range of tank length and the second is the deflection of the tank shall.
3. The local trim correction aft of midship has opposite sign to this of fore ship and therefore errors in measuring ballast tank placed along the whole hull length are neglect able.
4. The local trim correction may be important while fuel oil tank situated aft in engineer-room are measured. Also during bunker survey ("on hire" or "off hire" survey).
5. To simplify the procedure to apply the local trim correction it is suggested to calculate the trimming factor for each tank and place the result in sounding table on appropriate page for any tank and use formulas (10) and (11).

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Obliczenie strzałki ugięcia metodą długości względnej – przypadek ogólny

W praktyce znaki zanurzenia nie są umieszczane na pionach a ich odległości od pionów również nie są równe. Również środkowe znaki zanurzenia mogą znajdować się w pewnej odległości od owręża. Znak wolnej burty umieszczony jest zawsze na śródkręciu, a nie na owrężu.

W celu określenia współczynników paraboli odkształcenia nie stosując skomplikowanego rachunku macierzowego można postąpić następująco:

- ograniczyć się do obliczania paraboli zawartej między znakami rufowymi i dziobowymi przyjmując tę odległość (length between marks) $LBM = 1$;
- odległość środkowych znaków zanurzenia wyrazić w postaci ułamka w od znaku rufowego:

$$w = \frac{\frac{LBP}{2} - \Delta L_r}{LBM}$$

- podstawiając odcięte: 0 dla znaków zanurzenia na rufie, w – dla znaków zanurzenia na środku oraz 1 – dla znaków zanurzenia na dziobie otrzymujemy:

$$T_{KAR} = a \cdot 0^2 + b \cdot 0 + c$$

$$T_{KR} = a \cdot w^2 + b \cdot w + c$$

$$T_{KFR} = a \cdot 1^2 + b \cdot 1 + c$$

Rozwiązując ten układ równań otrzymujemy współczynniki a , b i c :

$$a = \frac{T_{KR} - T_{KAR} + w \cdot (T_{KAR} - T_{KFR})}{w^2 - w}$$

$$b = \frac{w^2 \cdot (T_{KFR} - T_{KAR}) - T_{KR} + T_{KAR}}{w^2 - w}$$

$$c = T_{KAR}$$

Zanurzenia na pionach:

$$T_{KA} = a \cdot \left(\frac{-\Delta L_r}{LBM} \right)^2 + b \cdot \left(\frac{-\Delta L_r}{LBM} \right) + T_{KAR}$$

$$T_{KF} = a \cdot \left(\frac{\Delta L_d + LBM}{LBM} \right)^2 + b \cdot \left(\frac{\Delta L_d + LBM}{LBM} \right) + T_{KAR}$$

Przełębienie:

$$t = T_{KF} - T_{KA}$$

Dla zanurzeń odnoszących się do pionów przyjmujemy $LBP=1$; $w=0,5$ i wówczas wzory na współczynniki przyjmą postać:

$$a = 2 \cdot T_{KA} - 4 \cdot T_K + 2 \cdot T_{KF}$$

$$b = 4 \cdot T_K - 3 \cdot T_{KA} - T_{KF}$$

$$c = T_{KA}$$

Odształcenie kadłuba między pionami:

$$d = T_K - \frac{T_{KF} + T_{KA}}{2}$$

W miejsce każdej wartości po prawej stronie tego wzoru podstawiamy jego równanie kwadratowe:

$$d = a \cdot \left(\frac{x'}{2}\right)^2 + b \cdot \frac{x'}{2} + T_{KA} - \left(\frac{a \cdot x'^2 + b \cdot x' + T_{KA} + a \cdot 0 + b \cdot 0 + T_{KA}}{2}\right) =$$

$$= \frac{2 \cdot a \cdot \left(\frac{x'}{2}\right)^2 + 2 \cdot b \cdot \frac{x'}{2} + T_{KA} - a \cdot x'^2 + b \cdot x' + 2 \cdot T_{KA}}{2} = a \cdot \frac{x'^2}{4} - a \cdot \frac{x'^2}{2} = -\frac{a}{4} \cdot x'^2$$

Jak wynika z końcowego rezultatu dla długości względnej między pionami kiedy ($x'^2=1$):

$$d = -\frac{a}{4} \quad (13)$$

Jeżeli obliczenia dotyczyły wielkości odczytanych na znakach zanurzenia to:

$$d = -\frac{a}{4} \left(\frac{LBP}{LBM}\right)^2$$

Natomiast wielkość odkształcenia w obrębie zbiornika wyniesie:

$$\Delta d = d \cdot \left(\frac{l_t}{LBP}\right)^2 \quad (14)$$